good agreement with experiment only in a narrow range in M_2/M_1 set by the region used. As M_2/M_1 increases, so does the discrepancy, which attains about 100% for $M_2/M_1 = 37$ (curve 1). From (1), one can obtain satisfactory agreement between the measurements and calculations on the Stanton number over a wide range in the mass ratio (curve 2).

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METHOD FOR DETERMINING THE GAS–DYNAMIC CHARACTERISTICS OF A NONEQUILIBRIUM HYPERSONIC FLOW OF NITROGEN BASED ON EXPERIMENTAL DATA ON THE STAGNATION PARAMETERS

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A method is developed for determining the pressure, temperature, density, velocity head, Mach number, and other gas–dynamic parameters of a nonequilibrium flow of nitrogen in a hypersonic nozzle. The method is based on experimental data on the stagnation parameters T_0, p_0, and p_0' and the gas–kinetic model adopted for nitrogen.

1. A method for determining the gas–dynamic quantities in the working part of gas–dynamic setups (GSs), based on the experimental data on the stagnation parameters of the gas T_0, p_0, and p'_0, as well as the known thermodynamic model of the given working gas, has been theoretically substantiated and is widely employed for operating regimes of GSs, in which the behavior of the working gases corresponds to the behavior of a perfect or equilibrium gas [1-5].

Thus, the gas–dynamic characteristics for the case of a perfect gas are determined from the relations [1]

$$\frac{p_0'}{p_0}=\left[\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}\right]^{\frac{\gamma}{\gamma-1}}\left(\frac{\gamma+1}{2\gamma M^2-(\gamma-1)}\right)^{\frac{1}{\gamma-1}},$$
$$\frac{q}{p_0'}=\frac{\gamma}{\gamma+1}\left[\frac{4\gamma}{(\gamma+1)^2}\right]^{\frac{1}{\gamma-1}}\left(1-\frac{\gamma-1}{2\gamma M^2}\right)^{\frac{1}{\gamma-1}},\quad q=\frac{\rho u^2}{2}, \tag{1}$$
$$\frac{T}{T_0}=\left(1+\frac{\gamma-1}{2}M^2\right)^{-1},\quad \frac{p}{p_0}=\left(1+\frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma}{\gamma-1}}.$$

In the case of an equilibrium gas the method developed in [1-5] also permits finding quite simply the gas–dynamic characteristics, using some tabulated functions that depend on T_0, p_0, M, and the composition of the gas.

In reality, in high-enthalpy GSs the gas flow is a nonequilibrium flow. The gas–dynamic characteristics are not only functions of p_0, T_0, M, and γ, but they also depend on the shape and dimensions of the nozzle and on the kinetic model and composition of the gas.

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In the general case of nonequilibrium flows there is at the present time no method for determining the gas-dynamic characteristics in nozzles from data on measurements of T_0, p_0, and p'_0 analogous to the method employed for frozen and equilibrium flows. In each specific case the information about the flow in the GS can be obtained from computer calculations of the flow of a viscous nonequilibrium gas.

A method is proposed below for determining the gas-dynamic parameters of a nonequilibrium flow of nitrogen in nozzles. This method, like for the case of a nonequilibrium flow, reduces to quite simple algorithms for calculating the gas-dynamic characteristics of a perfect gas. A number of constraints must be introduced in order to construct the method.

Nitrogen, one of the gases that has been studied in greatest detail and whose thermodynamic and kinetic characteristics are well known, was chosen as the working substance to be studied.

The following range of stagnation parameters is studied:

$$1000\ \text{K} < T_0 < 3000\ \text{K},\ 10^5\ \text{Pa} < p_0 < 100 \cdot 10^5\ \text{Pa}. \tag{2}$$

Among all the nonequilibrium processes only vibrational relaxation is important.

For the temperature range indicated the vibrational relaxation time τ is quite well known and on the basis of the model of a harmonic oscillator can be approximated by a function of the translational temperature and the pressure ($\tau p = f(T)$) [6-8]. the computational results showed that for $T < 3000$ K the effects of anharmonicity can be neglected (the characteristic temperature of nitrogen $\Theta = 3350$ K, so that $\Theta > T$, and only several of the first vibrational levels are primarily populated).

For the conditions (2) the corrections to the gas-dynamic quantities due to density effects, according to the data of R. M. Sevast'yanov and N. A. Zykov (see, for example, [9]), for $1000\ \text{K} < T_0 < 3000\ \text{K}$ equal about 2% for the pressure and about 1% for the temperature. The lower limit for the pressures p_0 is determined by values at which the viscosity in the core of the flow has virtually no effect on the readings of the sensors p'_0 for the studied operating regimes of GSs (T_0, M, Re) [10].

In what follows only operating regimes of GSs (p_0, T_0, and M) in which the condensation of nitrogen is not important will be studied.

The method is constructed for a class of axisymmetric nozzles with a hyperbolic contour, for which the ratio of the transverse cross-sectional area F/F_* is related with the coordinate x along the nozzle axis by the relation

$$F/F_* = 1 + (x/l)^2,\ l = r_*/\text{tg}\,\varphi.$$

For this class of nozzles the complex $p_0 \ell$, characterizing the degree to which the state of the flow in the nozzle departs from equilibrium,is, together with T_0 and γ, a similarity parameter for the regimes (2) [11]. The class of similar flows in nozzles can be enlarged by enlarging the class of nozzle shapes, admitting similarity; the conical ($F/F_* = (1 + x/\ell)^2$) and flat ($F/F_* = 1 + x/\ell$) nozzles belong to this class [12]. We note that for typical GSs and their operating regimes (T_0, p_0, M) the vibrational degrees of freedom are frozen on the accelerating section of the nozzle, whose profile is usually nearly hyperbolic. In this case the results obtained for hyperbolic nozzles can be transferred also to shaped nozzles for the same values of T_0 and $p_0 \ell$ [12-14].

2. An important factor employed in constructing this method is the freezing of the vibrational degrees of freedom (see [11-14] for a discussion of the criteria for freezing). Here the flow, starting at some section F_1, can be regarded as a flow of perfect gas with an adiabatic index $\gamma = \text{const} = 1.4$. According to the method, after the collection of gas-dynamic parameters T_1, p_1, M_1, etc. corresponding to the section F_1 has been found the so-called "effective" stagnation parameters $\tilde{T}_0$, $\tilde{p}_0$, $\tilde{\rho}_0$, and the "effective" area of the minimal section of the nozzle $\tilde{F}_*$ can be determined from the formulas for a perfect gas:

$$\tilde{T}_0 = T_1\left(1 + \frac{\gamma-1}{2} M_1^2\right),\ \tilde{p}_0 = p_1\left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}},$$

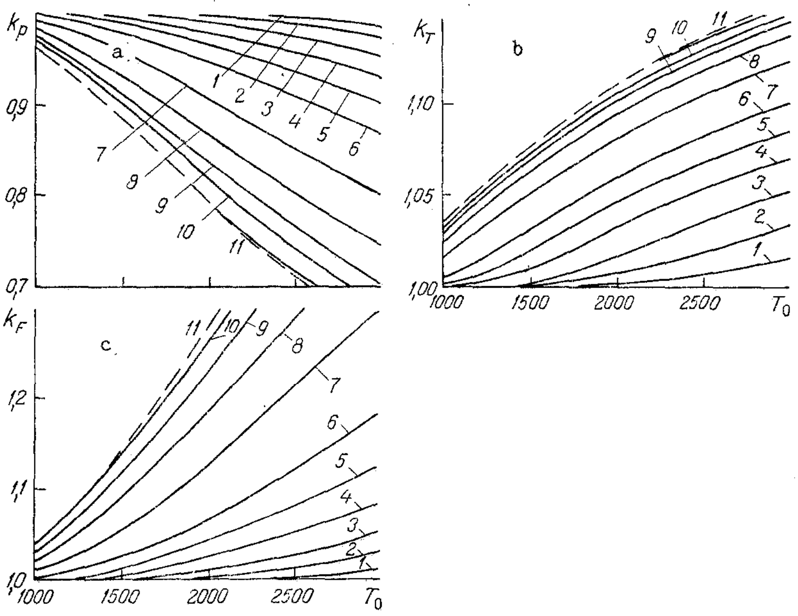

Fig. 1. The dimensionless coefficients k_p (a), k_T (b), and k_F (c) versus the stagnation temperature T_0 for different values of the parameter $p_0\ell$: 1) $p_0\ell = 3\cdot10^6$ Pa·cm; 2) 10^7; 3) $3\cdot10^7$; 4) 10^8; 5) $3\cdot10^8$; 6) 10^9; 7) 10^{10}; 8) 10^{11}; 9) 10^{12}; 10) 10^{13}; 11) ∞. T_0, K.

$$\tilde{\rho}_0 = \rho_1\left(1 + \frac{\gamma-1}{2}M_1^2\right)^{\frac{1}{\gamma-1}},$$

$$\tilde{F}_* = F_1\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} M_1\left(1+\frac{\gamma-1}{2}M_1^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}.$$

Thus the "effective" parameters correspond to the flow of a perfect gas, which for $F > F_1$ is thermodynamically identical to the flow of the nonequilibrium gas under study with the parameters T_0, p_0, and ρ_0 in the given nozzle $F(x)$. The proposed method can be used for shaped nozzles if the cross section F_1 is located in the accelerating part of the nozzle, i.e., on the section where the nozzle profile is nearly hyperbolic.

Let

$$\frac{\tilde{p}_0}{p_0} = k_p(T_0,\ p_0l),\quad \frac{\tilde{T}_0}{T_0} = k_T(T_0,\ p_0l),$$
$$\frac{\tilde{\rho}_0}{\rho_0} = k_\rho(T_0,\ p_0l),\quad \frac{\tilde{F}_*}{F_*} = k_F(T_0,\ p_0l). \tag{3}$$

Since for the regimes under study the molar mass μ = const, the coefficients k_i satisfy the relations

$$k_p = k_\rho k_T. \tag{4}$$

Unlike equilibrium flows the coefficients k_i are functions not of T_0, p_0, and M, but rather of T_0 and the parameter $p_0\ell$.

3. In this work the coefficients $k_i(T_0,\ p_0\ell)$ were determined based on analysis of the results of numerical integration of the system of differential equations describing nonequilibrium quasi-one-dimensional flow of nitrogen in a hyperbolic nozzle with the parameter $p_0\ell$ varying from values corresponding to a frozen flow ($p_0\ell = 0$) up to values corresponding to equilibrium flow ($p_0\ell = \infty$).

We shall formulate the gas-kinetic model of nitrogen for the range of T and p under study, in which the dissociation, condensation, intermolecular interaction forces, and the quantum effects of rotation of molecules can be neglected. We shall also assume that the translational and rotational degrees of freedom are in equilibrium with one another. These conditions satisfy the model of vibrational equilibrium. In the harmonic-oscillator approximation the system of gas-kinetic equations for a one-dimensional flow of nitrogen has the form [11]

$$\bar{\rho}\bar{u}\bar{F} = \text{const},\ \bar{\rho}\bar{u}\frac{d\bar{u}}{d\bar{x}} + \frac{d\bar{p}}{d\bar{x}} = 0,\ \bar{u}^2 + \frac{\bar{h}}{\bar{h}_0} = 1,\ \bar{\mu} = 1,\ \bar{p} = \frac{\bar{\rho}\bar{T}}{2\bar{h}_0},$$
$$\bar{h} = \frac{7}{2}\bar{T} + \frac{\Theta/T_0}{\exp(\Theta/T_\text{к}) - 1},\ \bar{u}\frac{d\bar{e}_\text{к}}{d\bar{x}} = \frac{\bar{e}_V(T) - \bar{e}_V}{\bar{\tau}}. \tag{5}$$

The system of equations (5) is written in dimensionless form. The variables were scaled as follows:

$$\bar{u} = \frac{u}{V_{max}},\ \bar{T} = \frac{T}{T_0},\ \bar{\rho} = \frac{\rho}{\rho_0},\ \bar{p} = \frac{p}{\rho_0 V_{max}^2},\ \bar{\mu} = \frac{\mu}{\mu_{N_2}},\ \bar{h} = \frac{h}{RT_0},$$
$$\bar{e}_V = \frac{e_V}{RT_0},\ \bar{x} = \frac{x}{r_*},\ \bar{F} = \frac{F}{F_*},\ \bar{\tau} = \frac{\tau r_*}{V_{max}}.$$

Here e_V is the specific vibrational energy of nitrogen, T_V is the vibrational temperature of nitrogen, μ_{N_2} = 28 kg/mole, $V_{max} = \sqrt{2h_0}$.

The specific form of the expression for the relaxation time τ is taken from [8]. The system of equations (5) was solved numerically by Gear's method [15]. The coefficients k_i (T_0, $p_0\ell$) obtained are presented in Fig. 1. The broken lines show the values of k_i for equilibrium flows ($p_0\ell = \infty$), obtained taking into account the anharmonicity of the vibrations (solid lines with $p_0\ell = \infty$), were found based on the formulas [16]:

$$k_T = 1 + \frac{\gamma - 1}{\gamma}\xi(\exp\xi - 1)^{-1},$$
$$k_p = k_T^{\frac{\gamma}{\gamma-1}}[1 - \exp(-\xi)]\exp\left(-\frac{\xi}{\exp\xi - 1}\right),\quad \xi = \frac{\Theta}{T_0}.$$

In the range of stagnation temperatures and pressures under study (2) the effect of anharmonicity is small. Thus for T_0 = 2500 K the difference in k_p with $p_0\ell = \infty$, caused by anharmonicity of the vibrations, will be about 0.15%; the difference in k_T will be about 0.1% (see Figs 1a and b).

4. If the flow in the shock layer near the nozzle is close to being frozen, which is typical for modern GSs [5], then the gas-dynamic characteristics of the flow M, T, p, etc. can be determined based on the formulas for a perfect gas (1), replacing the values of T_0 and p_0 by the values of $\tilde{T}_0$ and $\tilde{p}_0$, respectively.

In the general case the flow near the nozzle is nonequilibrium. We shall evaluate the maximum difference in the quantities p'_0 and other gas-dynamic characteristics caused by the fact that the flow is in a nonequilibrium state. It is shown in [17] that the largest difference in the readings of the sensor p'_0 will be observed with frozen (p'_{0f}) and equilibrium (p'_{0e}) shock layers in the case when the flow in the nozzle is an equilibrium flow. Thus, for example, the maximum difference in the readings of the sensor $\Delta = (p'_{0e} - p'_{0f})/p'_{0f}$ equals 1.3% for T_0 = 3000 K and about 1% for T_0 = 2000 K (naturally, the comparison is made for the same state of the gas in the working part of the GS).

The maximum error in determining the gas-dynamic variables for the range of stagnation parameters studied, taking into account the uncertainty in the state of the gas in the shock layer in front of the sensor, does not exceed 0.3% for M, 0.5% for T, 1.8% for p, and 1.3% for ρ and q. Here the fact that for a perfect gas with an adiabatic index γ = 1.4 at hypersonic velocities

$$\frac{d\ln M}{d\ln p_0'} \approx 0,2,\quad \frac{d\ln T}{d\ln p_0'} \approx 0,4,\quad \frac{d\ln p}{d\ln p_0'} \approx 1,4,\quad \frac{d\ln q}{d\ln p_0'} \approx \frac{d\ln\rho}{d\ln p_0'} \approx 1$$

has been taken into account.

5. In summary, based on the results obtained in this work, the gas–dynamic parameters of a nonequilibrium flow of nitrogen are determined in the following order:

1) the experimental values of p_0, T_0, and p'_0 are determined;

2) for a given nozzle the parameters $\ell = r_*/\tan\phi$ and $p_0\ell$ are calculated;

3) the functions $k_p(T_0, p_0\ell)$, k_T, k_ρ and k_F are found based on Fig. 1 and the relations (4);

4) The "effective" stagnation parameters and the "effective" area of the minimum cross section of the nozzle are found from the formulas (3); and,

5) the gas–dynamic parameters sought are determined based on the formulas presented below for a perfect gas with $\gamma = 1.4$:

$$\frac{p_0'}{\tilde{p}_0} = \left(\frac{6M^2}{M^2+5}\right)^{3,5}\left(\frac{6}{7M^2-1}\right)^{2,5},\ T = \tilde{T}_0\left(1+\frac{M^2}{5}\right)^{-1},$$

$$p = \tilde{p}_0\left(1+\frac{M^2}{5}\right)^{-3,5},\ \rho = \tilde{\rho}_0\left(1+\frac{M^2}{5}\right)^{-2,5},\ F = \tilde{F}_*\frac{125\left(1+M^2/5\right)^3}{216M},$$

$$q = 0{,}5437p_0'\left(1-1/7M^2\right)^{2,5}.$$

In conclusion we call attention to the following: although the relations for one-dimensional flows of a nonviscous nonequilibrium gas were employed to determine the gas–dynamic parameters, the method can also be used for viscous non-one-dimensional flows, since the effect of the viscous boundary layer and the non-one-dimensionality of the flow are taken into account based on data on the distribution $p_0(x, y, z)$ (y0z is the plane perpendicular to the axis of the tube).

The only necessary condition is that the core of the flow, in which the flow can be non-one-dimensional, must be nonviscous. Locally, however, in each stream tube from the core of the flow the flow is controlled by the laws of nonviscous, one-dimensional motion. In using the proposed method for non-one-dimensional flows, however, we assume that for each i-th stream tube in the core of the flow the local value of the parameter $p_0\ell_i$ is the same and equals $p_0\ell$ ($\ell = r_*/\tan\phi$). In other words, the effect of the nonequilibrium nature of the flow for each stream tube is assumed to be identical. The quantities r_* and $\tan\phi$ are determined by the geometric characteristics of the nozzle without any corrections for the thickness of the boundary layer. This is justified by two facts: 1) in the region of the flow up to the cross section F_1 for typical operating conditions of the GSs the boundary layer at the nozzle walls is quite thin; 2) the effect of the parameter $p_0\ell$ on the gas–dynamic quantities in the working part of the nozzle is quite weak. As the calculations showed (see Figs. 1a and b), the maximum values with respect to $p_0\ell$

$$\frac{\partial \ln T}{\partial \ln(p_0 l)} \text{ and } \frac{\partial \ln p}{\partial \ln(p_0 l)}$$

are much less than unity (see, for example, [13, 14]).

NOTATION

p, pressure; T, temperature; ρ, density; u, velocity; q, velocity head; M, Mach number; γ, adiabatic index; h, enthalpy; e_V, vibrational energy; μ, molar mass; x, coordinate along the axis of the nozzle; F, transverse cross-sectional area of the nozzle; r, radius of the cross section of the nozzle; ϕ, half-angle of the asymptotic cone; Re, Reynold's number; θ, characteristic temperature of nitrogen; R, universal gas constant; and τ, relaxation time. The indices 0 and * refer to stagnation parameters and the conditions in the minimal cross section of the nozzle, respectively.

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COOLING OF CRYOAGENTS BY THE PUMPING-OUT OF VAPOR

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The cooling of cryogenic liquids by the pumping-out of vapor is considered theoretically, taking account of the variability of the thermodynamic properties, the heat fluxes to the cryostatting zone, and the specific heat of the vessel walls.

Cooling of cryogenic liquids by the pumping-out of vapor is widely used in outfitting sublimational cold-storage units (SCSU) [1], obtaining slushlike cryoagents [2], and simply for reduction in temperature level. In the evaporation of liquid to replace the vapor which has been removed, the remaining mass of liquid is cooled, on account of the latent heat of vaporization, and also as a result of the work done by the vapor on emission from the container.

In designing cooling systems in which pumping-out of vapor is employed, as well as the maintenance posts of SCSU [3], it is necessary to have information on the cryoagent losses on cooling to a specified temperature. An approximate method of calculating the cooling was outlined in [1], under the following assumptions: that the process is equilibrium (the liquid and vapor temperatures are equal to the saturation temperature at the given pressure); the latent heat of vaporization is constant; the heat fluxes to the system are negligibly small in comparison with its heat content; consumption of cryoagent in cooling the cryostat may be neglected; the mass rate of pumping-out is constant. In [4], the cooling of helium on pumping out its vapor was calculated, taking account of the variability of its properties and the external heat fluxes. However, constant mass rate of pumping-out and negligibly small specific heat of the cryostat was assumed. In [5], the cooling of nitrogen, oxygen, neon, and para-hydrogen by pumping-out was calculated, under assumptions analogous to those in [1]. Simple approximations were used for the dependence of the specific heat and the latent heat of vaporization on the temperature, which means that it is possible to obtain simple analytical computational relations.

The present work is devoted to theoretical investigation of the cooling of widely used cryogenic liquids — hydrogen, oxygen, nitrogen, argon, and methane — by the method of evacuating the vapor space, taking account of the variability of their properties, the heat fluxes in the cryostatted zone, and the specific heat of the cryostat walls.

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